CHAPTER 3

Making Maths Make Sense

> *'For the things we have to learn before we can do them, we learn by doing them, e.g. men become builders by building and lyre players by playing the lyre.'*
>
> – Aristotle, 330 BC[1]

In the previous chapter, we saw how the Accelerating Learning for All process upends traditional methods of literacy instruction. ALfA puts the student at the centre rather than the teacher, asks questions rather than giving answers, and moves from pictures-to-sounds-to-letters rather than rote memorising the letters before knowing how their sounds fit into words.

Similarly, ALfA numeracy progresses from known to unknown and concrete to abstract, using question-focused paired activities to facilitate deeper conceptual understanding before doing any written work. In this chapter, we'll show you how it's done.

First, we debate a crucial question – what, if anything, is wrong with the traditional way of teaching maths? Then, we show how an activity-driven process can be used across a range of topics, progressing from first steps like counting and number sense to long division and fractions.

What's Broken?

When we ask people what subject they love most, we get quite a variety of responses: Language, Science, Physical Education, etc. Yet when we ask what their least favourite subject is, most children (and adults) unite in choosing Maths.

Let's understand some of the fundamental reasons why the traditional approach to teaching maths isn't working for many learners:

- Maths teachers drill formulae and techniques until they're automatic for the students. But many children don't understand the reasoning behind the drills; they only do it that way because they were told to, so the work feels arbitrary.
- Maths involves many abstractions. These abstractions can be challenging for learners who don't yet have a conceptual understanding.

- Long lists of number problems appear disconnected from lived experience and irrelevant to the real world.
- Most maths work is individualised, so children do not interact with other students.
- Maths is usually taught via an inflexible process, leaving no room for creativity.

A central issue is that the industrial education system doesn't foster number sense: the feel for what numbers mean and how they apply to the real world. Number sense involves skills like understanding how much smaller one number is than another or being able to approximate about how large an arithmetic result will be before you calculate it. Many children in the industrial education system memorize formulae and can apply the steps to solve abstract problems, but they don't understand the concepts underlying the rules, and thus are unable to relate to maths in their daily lives.

As a result of these factors, a substantial proportion of students are bored and alienated by maths. Unsurprisingly, boredom has a negative effect on learning. John Hattie's meta-analysis of over 200 factors that impact learning found that students who felt bored suffered the most learning loss of all students without a disability like deafness or ADHD.[2]

Reliance on traditional math instruction has led to remarkable shortcomings in numeracy skills in many regions. For example, consider India's Annual Status of Education Report (2022). This survey found that over half of Indian students are over three years behind their grade-appropriate maths level. For instance, just 50 per cent of children in Grade 5 can do a two-digit by two-digit subtraction (typically taught in Grade 2).[3] Only 45 per cent of children in Grade 8 can perform a three-digit by one-digit division (typically taught in Grade 4).

Table 2: Annual Status of Education Report, 2022

Std.	*Not Even 1–9*	*Recognise Number*		*Subtract*	*Divide*	*Total*
		1–9	*11–99*			
I	37.6	36.8	19.8	4.1	1.7	100
II	16.9	36.1	33.1	10.1	3.9	100
III	9.8	27.6	36.8	17.6	8.3	100
IV	5.8	20.2	35.3	22.9	15.9	100
V	3.7	14.6	31.8	24.3	25.6	100
VI	2.8	10.2	30.4	24.9	31.7	100
VII	1.9	7.3	28.3	24.7	37.8	100
VIII	1.6	5.2	25.5	23.1	44.6	100

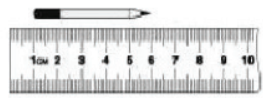
The problem isn't limited to government schools. By Grade 8, only 54 per cent of private school students in India can perform division. Furthermore, the ASER 2018 report showed that 40 per cent of students could already do those same division questions back in Grade 5.[4] In other words, only a quarter of the 60 per cent of students who failed to solve division problems in 5th grade had advanced in that subject four years later.

But it gets worse.

Most large-scale maths exams, including those conducted for ASER, test the ability to manipulate number problems without a deeper conceptual understanding. When more advanced exams are used to teststudents' ability to apply mathematical thinking practically, the results are shocking.

The WIPRO Quality Education Study (2011) tested the maths, science and language competencies of some 23,000 students of Grades 4, 6 and 8 from eighty-nine 'elite' private schools around India.[5] Let's look at how students did on some of those problems.

The length of this pencil is about_____.



A. 4 cm
B. 5 cm
C. 6 cm
D. 7 cm

Less than a third of Grade 4 students in elite schools got this question right. 67 per cent answered option C—not understanding that the placement of the base of the pencil needs to be considered when measuring it.

Anupam takes papers P and Q, which are exactly the same.

He crumples P as shown in the figure.

Which of the following statements about P and Q is true?

A. P has more weight than Q.
B. Q has more weight than P.
C. P and Q have equal weight.
D. P and Q both have no weight.

Remarkably, only 22 per cent of Grade 6 students in elite schools correctly answered **C**, that crumpling a paper makes no difference to its weight. A similar misconception occurred regarding a question about area, where only 45 per cent of Grade 6 students realised that cutting a paper in half and rearranging the two pieces would not change the area.

These problems are not unique to India. Even in a country as wealthy as the US, 29 per cent of young people have low numeracy skills.[6] Resources and staffing may differ from country to country, but the outdated pedagogy remains the same.

Aware that the traditional education system is proving inadequate, many education systems are in the process of trying various reforms. Some have advocated a move to algorithm-based calculation tips, such as 'Vedic Maths'.[7] Memorising formulae and tricks like these can help students solve test paper questions in some circumstances. However, such mental math aids don't conceptualise the subject and can't easily translate to solving real-world problems.

Others argue for more modern technology in education, extolling various apps that turn solving sums into a fun experience. We are all for gamification and interactivity in education. But access to technology remains limited in many poorer families; and for those who do have mobiles, the amount of time kids spend looking at screens is concerning! Further, many forms of on-screen education lack conceptual depth. Dressing up old pedagogies in new tech clothing doesn't change the fundamental difficulties in teaching mathematics.

More promising is the push for activity-based, hands-on maths learning. Sadly, though, these efforts at reform often end up as window dressing—fun activities that break the monotony of lessons but seldom connect conceptually to the lectures. A few extra bells and whistles on the horse and cart won't get us to an electric car! And the maths classroom needs much more than just an occasional activity to make it truly engaging and effective.

We believe that nothing in mathematics is inherently hard or scary. The difficulties originate only in how we teach it. You'll see in this chapter that maths can be the most concrete, relevant, and exciting subject your students will encounter when taught with a new approach.

The ALfA Numeracy Journey

ALfA's key insight is using the child's known experience to help them attain new knowledge. According to cognitive psychologist Nelson Cowan, 'New information must make contact with the long-term knowledge store in

order for it to be categorically coded.'[8] If we want learners to incorporate our lessons fluidly, we must use their long-term knowledge store as the foundation of the lesson.

As we saw in the last chapter, ALfA uses children's oral language knowledge and ability to recognise pictures as the foundational 'knowns' upon which to build up to 'unknowns' like letters. So, what mathematical knowledge do young children already possess? The answer is simple.

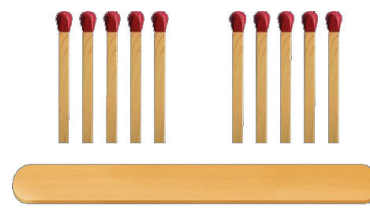
Counting.

ALfA maths starts from counting as a base and works up from there. Even learners who don't recognise numbers or symbols can still count items in their day-to-day lives. That's why we use tangible objects to teach simple operations. Only later will we connect these objects to abstract representations. Throughout the process, peer-learning activities facilitate the development of number sense.

Let's follow the journey of a numeracy learner using the ALfA approach.

Start with Real Things

We invite learners to begin by counting matchsticks. Only when a learner has proven the ability to count objects do we teach symbolic counting (having them make five tally marks on a page) and then abstraction (writing the number '5' to match the tally marks). Since our materials scaffold from step to step, students can proceed through the lesson in their learning pairs with minimal teacher intervention. From the first lesson, the programme also introduces children to concepts like 'less than' and 'more than'.



ALfA's techniques ensure that children develop conceptual understanding of numbers in every activity. For example, to teach two-digit numbers, we again use matches to represent 'ones' and then add ice cream sticks to represent 'tens'. We introduce this concept through tasks showing that ten matches are equivalent to one ice cream stick, then exercises which connect the matchstick / ice cream stick combinations to their written representation.

For larger numbers, bundles of ten ice cream sticks represent the '100s' column, and some other concrete object, like a spoon, can represent a thousand. (You may notice that all materials used in ALfA are cheap, reusable, sustainable and locally available.) Once again, children build up to these concepts by seeing that ten ice cream sticks (10 tens) are

equal to one bundle (100), and 10 bundles are equivalent to one spoon (1000). Thus, children learn to create four-digit numbers with a concrete representation via which they can easily visualise the differences between numbers, as opposed to written digits, which have no clear meaning to the early learner.

Math Games

Along with the concrete (physical objects), ALfA incorporates the kinesthetic (physical actions).

One fun activity is the 'switch out' game. One child lays out a few 1000s, 100s, 10s and 1s; then, their pair adds up the total to find the number represented. The first child changes one pile, and the second voices the new number quickly.

We've seen learners become remarkably fast at computing new numbers through games like these. Unlike written number games, where the real-world meaning of the numbers can get lost, these students remain connected to the numbers through their concrete representation. Teachers can devise numerous similar games from the same principles.

A group activity that bridges the concrete-abstract divide is the Ascending/ Descending Game. Each learner receives a card with a number written on it. They then arrange themselves into a line from smallest to largest (or vice versa). There are many variations you can play on this theme. For example, in an advanced version, the children have to stand further or closer together depending on how far apart their numbers are. Another idea is to give children cards with fractions on them, then ask them to arrange themselves likewise. As with most games, there are endless variations that the teacher—or better yet, the students—can come up with to teach different concepts.

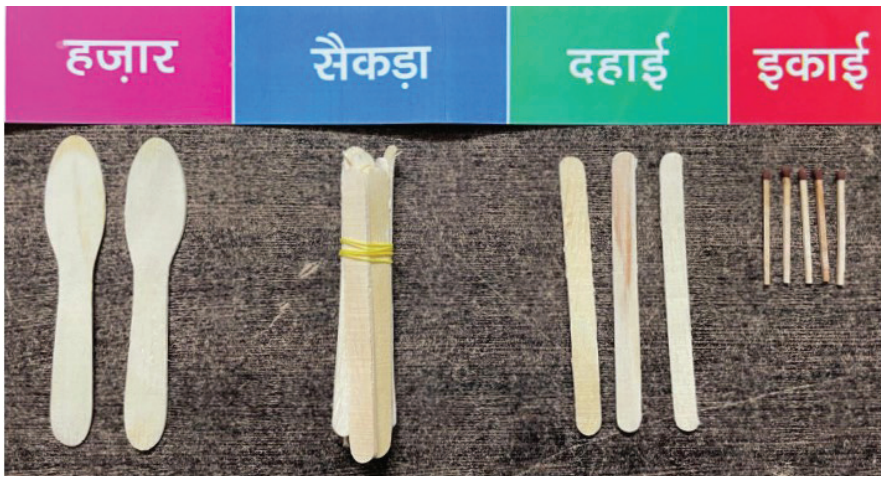
Even after they've mastered pictorial and abstract representation, students will continue to play math games. As in the literacy games, these activities heighten student interest while cementing concepts into their brain's cognitive structure. Children can play these games at home with their family, friends, or puppets. Playing games is much more engaging than doing homework, which often leads to a more significant learning impact. It even allows for greater creativity, as they use various available home materials. We've seen kids represent 'ones' using garlic cloves and 'tens' using onions!

All these games aim to help students develop their number sense and an intuitive grasp of quantity. Number sense is vital to building foundational numeracy.

Arithmetic Operations

Once children grasp the meaning of numbers, the programme continues into operations, such as addition, subtraction, multiplication, and division. Teachers apply the same procedure, starting with physical objects and only later bringing in abstractions.

We introduce the concept of a 'number bank' early on to help with arithmetic. In a two-digit number bank, students tape a blank paper on the desk and then draw a line down the middle. They arrange their matchsticks (ones) neatly on the right side and ice-cream sticks (tens) on the left side.

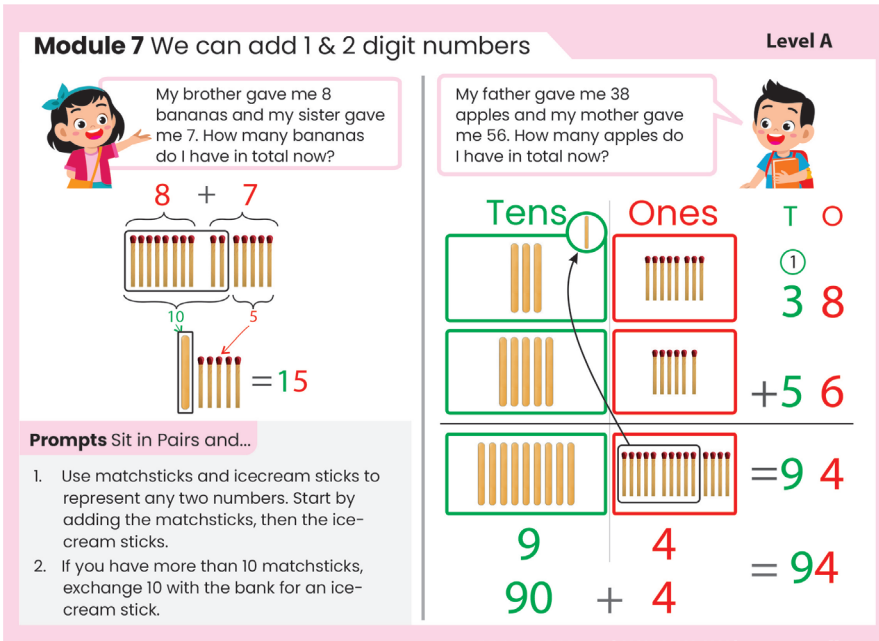


A 4-digit number bank

Suppose we are teaching addition. We start with a word problem so that students can connect the skill with their daily lives. But instead of solving the problem with written numbers, the learners represent their work using matchsticks and ice cream sticks. To start, the children pull the appropriate number of matchsticks and ice cream sticks out of the bank to form the number(s).

Consider the question below:

Child A pulls 38 out of the bank (three ice cream sticks and eight matchsticks), and Child B pulls out 56 (five ice cream sticks and six matchsticks). They work first with the ones: grouping the matchsticks yields fourteen. Since the children already understand that ten matches are equivalent to one ice cream stick, they can put ten matches into the bank and take an ice cream stick in exchange. This is a crucial rule to emphasise: The bank only allows you to exchange things of equal value.



Addition module: Solving word problems with concrete objects

Next, they work with the tens. Grouping the ice cream sticks together yields nine: five and three that they started with, plus the one they exchanged 10 matchsticks for. The pair now have nine ice cream sticks and four matches, or 94.

In the traditional education system, children taught to perform a 'carry the 1' adding operation often learn how to do it without understanding what it means. Those problem sets are divided into 'non-carry-over' and 'carry-over' sections, which drill the techniques without promoting comprehension.

The problem with this arbitrary distinction was driven home to us by one student who, when we gave them a sum to solve, asked, 'Is it a carry-over question, sir?' When we said 'yes', they were able to solve it. But the fact that they had to ask indicates that they had no idea what the process of carrying over meant or why it is employed in some problems and not others.

In ALfA, we introduce questions with and without carry-over simultaneously. As with all ALfA exercises, children make questions for each other randomly, not knowing ahead of time whether the solution will require a carry-over or not. Working through several of these exercises using physical objects will give the children an excellent intuitive grasp of carrying. Then, when the children graduate to abstract representation,

they will know what they are doing and why it works, rather than just following rules they don't understand.

One final benefit of the concrete-to-abstract methodology is that it enables children to visualise and solve even large sums. Memory masters tell us that the key to remembering large amounts of data is to visualise it. What better way to teach visualisation than to drill with manipulable visuals from the beginning? With practise, even Grade 2 children can solve four-digit addition problems quickly and accurately without needing to work them out on paper.

Demonstrating Competence: Story Sums

As learners advance, employing objects or tallies is no longer necessary. However, we never stop using real-life examples and word problems to teach concepts, as we don't want students to lose track of the relevance of their work.

Our keynote word problem format is the 'story sum'. Story sums are a single maths story that operates as an entry gate to many more such stories, building a template for infinite questions. Here is an example:

> *At 6:45 am, Maria left for the market with 200 rupees. The market she walked to was 0.75 kilometres away, and she wanted to get there right when they opened at 7 am. When Maria arrived, she saw eggs were 80 rupees/dozen, milk was 50 rupees a litre, and bread was 36 rupees per loaf, so she bought half a dozen eggs and three loaves of bread. Unfortunately, she had to wait behind four people in line and stayed ten more minutes. When Maria returned home, she divided the change among her three kids.*

This short story can cover a range of concepts—addition, subtraction, multiplication, division, decimals, fractions, time, distance, speed, and money. The ALfA workbook suggests some questions for children to solve based on the story, such as:

- How much money did Maria spend?
- What was the cost per egg?
- How fast did Maria walk to reach the market right when it opened?
- On average, how long did the people in front of Maria take to purchase groceries?
- How much money did each of her children receive?

After students have solved these questions in pairs, they can make up more such questions for each other based on the same story. Not all of the questions children come up with will be answerable. That teaches

an additional lesson relevant to real-world problems—learning to tell whether you have enough information and when you need more.

The story sum is a template from which students can create many more stories for each other. They can revise each situation by changing the names, numbers, and type of store. Sometimes, they adjust their questions to make them easier or harder for their partner.

Here's another example:

Grandma is planting a garden. Her yard is 12 metres long and 5 metres wide, and she wants to use half of the yard for the garden. Grandma must build a fence around her garden to protect it from animals. She will fertilise the garden with cow manure purchased from a farmer. The fencing costs 600 nairas per meter, and the cow manure costs 10 nairas per litre. She thinks she will use about five litres of manure per square meter of the garden. Grandma has divided the garden between spinach, tomatoes, and chilli peppers. The seed costs and production of these different options are:

	Seed Costs (Nairas/Bag)	*Productivity (kg/m²)*
Spinach	80	5
Tomato	120	10
Chilli pepper	50	1

Again, the story sum connects with real-life experiences for students – of farming, grandparents, and making decisions with trade-offs. The number of questions students can derive from this story is enormous. Still, some of those questions would require additional information to answer. An inquisitive child may ask the teacher follow-up questions such as 'How large an area can you plant with one bag of seeds?' or 'Does the garden have to be rectangular?'

The story integrates many mathematical concepts – including perimeter, area, volume and money. It goes beyond maths to connect with other subjects, like science ('Why do different crops have varying productivity?') and environmental studies ('Why is grandma using cow manure rather than synthetic fertiliser?').

Even when your students advance to higher-level math, it is essential to keep using exercises that are relevant and interesting to the learner. The purposes are manifold. Children solving such problems engage in more critical thinking than when solving a simple formula. They are more engaged than with abstract problems on a worksheet. They can see the significance of their work, more easily connecting it to practical tasks outside the classroom.

When taught with concrete objects, fun activities, and self-directed real-world problems, numeracy skills no longer need to be boring or scary. Instead, they can be an absolute joy to learn.

A Year's Numeracy in 45 Days?

Recall that half of Grade 5 students in India can't do a two-digit subtraction. In this light, the notion that a student can go from ground zero to functionally numerate in just 45 working days might seem too good to be true. However, the ALfA process, scaffolding neatly from students' existing knowledge and engaging them with paired activities, makes this rapid progress possible.

This video, shot in Shamli district, showcases the children's impressive maths gains within forty-five days. Read more about the Shamli ALfA project in Chapter 10.

Scan the QR or go to www.learn.literacynow.app to see the ALfA numeracy book and understand how it scaffolds from simple to complex.

Now that you understand how ALfA concepts facilitate activity-based learning in language and math, you may wonder how these techniques would transfer to science, social studies, and other subjects. The ALfA process moves easily across various domains, allowing subjects to be integrated into the child's mind rather than remaining in separate silos. The key is paired learning. As we'll see in the next chapter, teachers of any subject can utilise paired learning to introduce material and investigate their world in new and exciting ways.